Understanding Theory X via Discrete Light-Cone Quantization

$R = \langle V_+, V_- \rangle$

Goal: Describe $X_{A_{N-1}}[\mathbb{R}^4 \times C_R]$ where $C_R = \mathbb{R}^{1,1}/\Lambda_R$, Λ_R a null-lattice $\cong \mathbb{Z}$

(Compactification of theory X on a light-like circle). Using inspiration from Matrix Models.

$\mathrm{Span}\{X_+, X_-\}$

X_- compact

Recall

$$X_{A_{N-1}}[W^{5,1}] = \begin{array}{c}\text{M5}\\ \text{World-Volume}\\ \text{EFT}\end{array}\left\{ M[\underbrace{W^{5,1}}_{N \text{ M5-branes}} \times \mathbb{R}^5] \right\}$$

May not always be a product, more gen.

$X_{A_{N-1}}[\text{World-Vol.}]$

Where (c.f. nLab)

```latex
\begin{tikzcd}
\begin{array}{c}\text{M2}\\ \text{M5 branes}\end{array} \subset M \arrow[r, "\text{Low Energy}", "\text{EFT}"'] \arrow[d] & \text{11-D SUGRA} \arrow[d] \\
\text{String-Theory} \arrow[r, "\text{Low Energy}", "\text{EFT}"'] & \text{10-D SUGRA}
\end{tikzcd}
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$l_s = (\alpha')^2 = g_s^{-1/3} l_p$

In particular

11-direction

$$M[\mathbb{R}^{9,1} \times S^1_R] \underset{\substack{\text{Reduction}\\ \text{(in SUGRA limit)}}}{\overset{\text{KK}}{=}} \text{IIA}\left[\mathbb{R}^{9,1} ; g_s = \left(\frac{R}{l_s}\right)^{2/3}\right]$$

To See how this reduction works go to the SUGRA limit; in particular we Find

$$(g_{\mu 11})_{\mu=1}^{10} \longrightarrow A \in \Omega^1(\mathbb{R}^{9,10}) \quad U(1) \text{ Gauge Field}$$

$$\text{Object w/ } P_{11} = \frac{N}{R} \longrightarrow \hat{N} \quad (U(1) \text{ electric Charge})$$

Super-graviton States
with $\hat{P}_{11} = NR$, $\longrightarrow$?
$N \in \mathbb{Z}$ $\hat{p}^\perp = 0$.

Answer : ? = $\overset{\text{BPS}}{\text{Bound States of N D0 branes}}$ ~ Non-perturb. objects in IIA

<u>Check</u>:

- D0 branes are the only IIA states with charge under U(1)
- Supergravitons $\xrightarrow[\text{reduction}]{\text{KK}}$ Tower of massive KK particles charged under U(1) :

$$m^{KK} = \frac{N}{R}, \quad \hat{Q} = \frac{N}{R}, \quad N \in \mathbb{Z}$$

BPS Saturated

Spectrum matches D0 brane spectrum $(N=1)$

BPS bound state of N D0 branes $(N>1)$.

<u>Rough Claim</u> :

" $M[\mathbb{R}^{9,1} \times S^1_R]$ in " " $\longleftrightarrow$ " World volume theory
Infinite Mom. Frame on D0 branes
(QM)

<u>Infinite Mom. Frame (IMF)</u>

- Boost to a frame so that all P_{11} momenta of any state under consideration are large and positive
- $\hat{P}_{11} \neq 0$ sector decouples from $\hat{P}_{11} = 0$ sector :

$$E = \sqrt{p^2 + m^2}$$

$$= \sqrt{P_{11}^2 + P^2 + m^2}$$

$$= P_{11} + \frac{P^2}{2P_{11}} + \frac{m^2}{2P_{11}} + \mathcal{O}\left(\frac{1}{P_{11}^2}\right)$$

$\Rightarrow$ relative energy between a $P_{11} = A \neq 0$ state and $P_{11} = 0$ state is $\mathcal{O}(A)$ but we can take $A \to \infty$ $\Rightarrow$ decoupling.

$\Rightarrow$ Only States with $\hat{P}_{11} \geq 0$ are relevant in the (limiting) IMF.

$\Rightarrow$ Only D0 branes remain

(and open strings connecting D0 branes $\longleftrightarrow$ D0 brane bound states)

$\Rightarrow$ Study QM of bound states of N D0 branes, $\hat{P}_{11} = \frac{N}{R}$, then let $N \longrightarrow \infty$.

Dim red. of 10 D, $\mathcal{N}=1$ SYM to D0 brane world volume (Witten: Bound states of p-branes)

$$L = \frac{1}{2g_s}\left\{\sum_{i=1}^{9} \mathrm{Tr}\, \dot{X}^i \dot{X}_i + 2\theta^T\dot{\theta} - \frac{1}{2}\sum_{i,j=1}^{9} \mathrm{Tr}\,[X^i, X^j]^2 - 2\theta^T \sum_{i=1}^{9} \gamma_i [\theta, X^i]\right\}$$

$\uparrow$ $A=0$ Gauge $\qquad \mathfrak{u}(n)$

Where

(Skew-Hermitian)

$X^i \in \mathrm{Mat}_{N\times N}\,\mathbb{C}$ (adjoint of $U(N)$), $i=1,\ldots,9$

$\theta \in \mathrm{Mat}_{N\times N} \otimes$ (Spin(9) rep.)

$\uparrow$ 16 component spinor

$g_s = \frac{R}{\ell_s}$

low energies $\longleftrightarrow$ "large X" $\Rightarrow \mathrm{Tr}\,[X^i, X^j]^2 = 0 \Rightarrow$ Simultaneously diagonalizable. Diagonal entries of $X^i \longleftrightarrow$ ith coordinates of N D0 branes.

<u>Decompactification Limit</u>

Need $P_{11} = \frac{N}{R} \longrightarrow \infty$

$N \longrightarrow \infty$

$$\Rightarrow \quad N \longrightarrow \infty, \quad R \longrightarrow \infty, \quad \frac{N}{R} \longrightarrow \infty$$

Study only processes with states whose energies vanish at least $\mathcal{O}\left(\frac{1}{N}\right)$ (other states have ∞-large energy).

Conclusion:

$M[\mathbb{R}^{9,1} \times S^1_R]$ in IMF $=$ $U(N)$ matrix model in the limit that $N \to \infty$

<u>DLCQ Refinement</u> (Susskind: hep-th/9704080)

Let $C_R = \mathbb{R}^{1,1}/\Delta$ as in intro, then

(*) Total Momentum N Sector of $M[\mathbb{R}^8 \times C_R]$ $\mathcal{H}^{M[\mathbb{R}^8 \times C_R]}\left(\hat{P}_- = \frac{N}{R}\right)$ $=$ (Quantization of $U(N)$ matrix model) $\mathcal{H}^{U(N)\text{-mat}}$

$\longleftrightarrow$ Representations of 11-D Gallilean Group w/ 16 SUSY (broken from 32 due to compactification and BPS presence)

<u>DLCQ of Theory $X_{A_{N-1}}$</u>: World Volume-theory on M5 branes

Same process, but in the presence of N coincident M5 branes:

D4 branes in IIA language $\longleftrightarrow$ D0-D4 bound states in IMF

Berkooz & Douglas: 9610236 Fivebranes in M(atrix) Theory:

Use language of $\mathcal{N}=(1,0)$, $d=6$ SUSY ($\swarrow$ 8-SUSY) (dimensionally reduce to D0 brane worldvolume via previous arguments): RHS of (*) w/ K M5 branes is

$$\underbrace{\text{1 Hypermultiplet in } \mathrm{Adj}_K + \text{VM of } U(K)}_{\text{Usual M(atrix) Theory (D0-D0) strings}} + K \text{ Hypermultiplets in Fundamental } K \,\&\, \overline{K} \text{ of } U(N)$$

$\frac{1}{2}$ hypers

(One Fund. per D0-D4 String)

There are 8 Supercharges:

$$8 = \frac{1}{2} \cdot \frac{1}{2} \cdot 32$$

Restricting to Sector w/ $P_- = \frac{N}{R}$

BPS M5 brane

To get $X_{A_{K+1}}[\mathbb{R}^4 \times C_R]$ we look at the low energy effective field theory (should be M5 brane worldvolume theory) in the limit that gravity decouples: $M_p \to \infty \Rightarrow l_p \to 0 \Rightarrow g_s = g_{QM} = M_p^6 R^3 \to \infty$.

LEEFT: σ-model into $\mathcal{M}_{vac} \supset \mathcal{M}_{Higgs}, \mathcal{M}_{Coulomb}$

at sing. of Higgs branch

$\mathcal{M}_{Coulomb}$ Zero HM VEV (non zero VM VEV) $\longleftrightarrow$ (Encodes) Transverse positions of D0 branes from K D4 branes

$\mathcal{M}_{Higgs}$ (zero VM VEV) $\longleftrightarrow$ Bound State of D0 brane lives on D4-brane

$\updownarrow$

Moduli of N-instanton in $U(K)$ gauge theory on D4 brane

$g_{QM} \to \infty$ limit: Higgsed VM gain infinite masses $\Rightarrow \mathcal{M}_{Coulomb}$ "decouples" from the interior of the Higgs branch

Metric on $\mathcal{M}_{Higgs}$: independent of g_{QM}

- Gains no Quantum Corrections (non-renorm. Thm: hep-th/9603042 Argyres, Plesser, Seiberg)

$\mathcal{M}_{Higgs}$: Let adjoint HM Scalars be: $X, \tilde{X} \in \mathrm{Mat}_{N\times N}\mathbb{C}$

HM Scalars : $q^i \in N$
$\tilde{q}^i \in \overline{N}$

Then

$$\mathcal{M}_{Higgs} = \left\{ \begin{array}{c} [X, X^\dagger] - [\tilde{X}, \tilde{X}^\dagger] + q_i q_i^\dagger - (\tilde{q}^i)^\dagger(\tilde{q}^i) = 0 \\ [X, \tilde{X}] + q_i \tilde{q}^i = 0 \end{array} \right\} \Big/ \text{Gauge}$$

$\| $ ADHM

$$\mathcal{M}_N[U(K)] \cong \underset{\substack{\uparrow \\ \text{Param. by } Tr(X),\ Tr(\tilde{X})}}{\mathbb{R}^4} \times \mathcal{M}^\circ_N[U(K)]$$

$\dim_{\mathbb{C}} \mathcal{M}_H = 4NK$ (HK manifold)

Quantization Gives:

$$\hat{P}_+ = \hat{H} = \frac{R}{N} g^{\alpha\beta}_{\mathcal{M}_H} \hat{\Pi}_\alpha \hat{\Pi}_\beta + \text{Ferm.} \quad \alpha, \beta \in 1, \ldots, 8NK$$

$$\hat{P}_- = \frac{N}{R} = \frac{1}{P_-} \cdot g^{\alpha\beta}_{\mathcal{M}_H} \hat{\Pi}_\alpha \hat{\Pi}_\beta$$

Included For Convenience $\left(\hat{H} = \hat{P}_+ = \dfrac{P_i^2 + M^2}{P_-}\right)$.

$$\text{Global Symmetries: } \underbrace{SU(2)_R \times SU(2)_L}_{\text{rotation Symmetries inside 5-brane and } \perp \text{ to Light-cone Coord} - (Spin(4))} \times Spin(5) \times U(K)_{global}$$

rotations $\perp$ to 5-brane = R-Sym of 6D Theory X

Bosonic Space-time Symmetries preserving $\hat{P}_- = \frac{N}{R}$ Sector: (Commuting with $\hat{P}_-$)

Gallilean generators $\underset{\substack{\uparrow \\ \text{rotations}}}{M_{ij}}$, $\underset{\substack{\uparrow \\ \text{translations}}}{P_i}$, $H = P_+ = P_0 + P_1$; $V_i = \underset{\substack{\uparrow \\ \text{boosts}}}{M_{0i} - M_{1i}}$

Preserved Conformal Gens: $\underbrace{\underset{\substack{\uparrow \\ \text{Special Conf.}}}{K_- = K_0 - K_1}, \quad T = \underset{\substack{\uparrow \\ \text{Dilitation}}}{D} - \underset{\substack{\uparrow \\ \text{Boost in } X^1 \text{ direction}}}{M_{01}}}_{\shortparallel}$

$$\text{Centralizer}_{SO(2,6)}\,[\hat{P}_-]$$

In our model

K, T = Combinations of derivatives of Kähler potential K and $\underset{\nwarrow}{\hat{\Pi}}$-ops.

Where
$$K = \mathrm{Tr}\left[q_i^+ q^i + (\tilde{q}^i)^+ (q_i) + X^+ X + \tilde{X}^+ \tilde{X} \right]$$

Fermionic:
- 8 Spacetime SUSY "Q"s
- 8 Superconformal generators "S"'s

Need for resolution of Singularities: Want to make sense of

State $\longleftrightarrow$ op Correspondence

but all Primary States: $K_-|S\rangle = 0$, are localized at the Origin; a singular point. Want to make sure there are no hidden degrees of freedom at the origin.

Resolution: • Add FI term (QFT P.O.V.)

$\updownarrow$

Turn on 3-Form Field (M-theory) in 11D.

Resulting Theory is not Theory X, but Flows to it in the IR.

not even Lorentz invariant

(FI param ζ acts as UV cutoff.)

- Did some computations for cases $N=1, K=2$, $N=2, K=1$ where $\mathcal{M}_{1+} = \mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2$, Resolution is usual blowup of $\mathbb{Z}_2$ singularity.

Refs: • Aharony - Berkooz - Kachru - Seiberg - Silverstein : hep-th/9707079

- A B - Seiberg : 9712117
- Banks - Fischler - Shenker - Susskind : 9610043
- Susskind : 9704080
- Berkooz - Douglas : 9610236
- Witten : 9510135