

# Understanding Theory X via Discrete Light-Cone Quantization

$$R = \langle v_+, v_- \rangle$$

Goal: Describe  $X_{A_{N-1}} [R^4 \times C_R]$  where  $C_R = \mathbb{R}^{1,1} / \Lambda_R$ ,  $\Lambda_R$  a null-lattice  $\cong \mathbb{Z}$

(Compactification of theory X on a light-like circle). Using inspiration

From Matrix Models.

$$\text{Span} \left\{ X_+, X_- \right\}$$

Compact

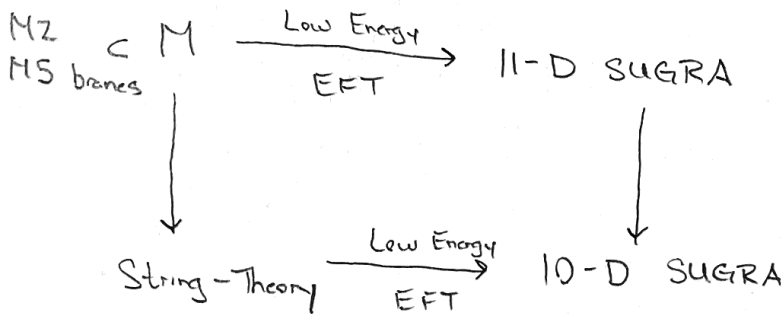
Recall

$$X_{A_{N-1}} [W^{5,1}] = \text{MS World-Volume EFT} \left\{ M [W^{5,1} \times \mathbb{R}^5] \right\} X_{A_{N-1}} [\text{World-Vol.}]$$

N MS-branes

May not always be a product, more gen.

Where (c.f. nLab)



$$l_s = (\alpha')^2 = g_s^{-1/3} l_p$$

In particular

$$M [\mathbb{R}^{9,1} \times S^1_R] \xrightarrow{\text{KK Reduction (in SUGRA limit)}} IIA [\mathbb{R}^{9,1}; g_s = \left(\frac{R}{l_s}\right)^{2/3}]$$

11-direction

To see how this reduction works go to the SUGRA limit; in particular

we find

$$\left( g_{\mu\nu} \right)_{\mu=1}^{10} \longrightarrow A \in \Omega^1(\mathbb{R}^{9,10}) \text{ U(1) Gauge Field}$$

$$\text{Object w/ } P_{11} = \frac{N}{R} \longrightarrow N \text{ (U(1) electric charge)}$$

Super-graviton States

$$\text{with } \hat{P}_{11} = NR, \longrightarrow ?$$

$$N \in \mathbb{Z}, \hat{P}^\perp = 0.$$

Answer: ? = <sup>BPS</sup> Bound States of N D0 branes ↙ Non-perturb. objects in IIA

Check:

- D0 branes are the only IIA states with charge under  $U(1)$

- Supergravitons  $\xrightarrow[\text{reduction}]{\text{KK}}$  Tower of massive KK particles charged under  $U(1)$ :

$$m^{\text{KK}} = \frac{N}{R}, \quad \hat{Q} = \frac{N}{R}, \quad N \in \mathbb{Z}$$

↖ ↗  
BPS Saturated

Spectrum matches D0 brane spectrum ( $N=1$ )

BPS bound state of N D0 branes ( $N > 1$ )

Rough Claim:

"  $M[\mathbb{R}^{9,1} \times S^1_R]$  in " ↔ " World volume theory on D0 branes (QM) "

Infinite Hom. Frame on D0 branes

Infinite Hom. Frame (IMF)

- Boost to a Frame so that all  $P_{||}$  momenta of any state under consideration are large and positive

- $\hat{P}_{||} \neq 0$  sector decouples from  $\hat{P}_{||} = 0$  sector:

$$\begin{aligned} E &= \sqrt{p^2 + m^2} \\ &= \sqrt{P_{||}^2 + P^2 + m^2} \\ &= P_{||} + \frac{P^2}{2P_{||}} + \frac{m^2}{2P_{||}} + \mathcal{O}\left(\frac{1}{P_{||}^2}\right) \end{aligned}$$

$\Rightarrow$  relative energy between a  $P_{||} = A \neq 0$  state and  $P_{||} = 0$  state is  $\mathcal{O}(A)$  but we can take  $A \rightarrow \infty \Rightarrow$  decoupling.

$\Rightarrow$  Only states with  $\hat{P}_{11} \geq 0$  are relevant in the (limiting) IMF.

$\Rightarrow$  Only D0 branes remain

(and open strings connecting D0 branes  $\longleftrightarrow$  D0 brane bound states)

$\Rightarrow$  Study QM of bound states of  $N$  D0 branes,  $\hat{P}_{11} = \frac{N}{R}$ , then let  $N \rightarrow \infty$ .

Dim red. of 10D,  $N=1$  SYM to D0 brane worldvolume (Witten: Bound states of p-branes)

$$L = \frac{1}{2g_s} \left\{ \sum_{i=1}^9 \text{Tr} \dot{X}^i \dot{X}_i + 2\theta^T \theta - \frac{1}{2} \sum_{i,j=1}^9 \text{Tr} [X^i, X^j]^2 - 2\theta^T \sum_{i=1}^9 \gamma_i [\theta, X^i] \right\}$$

Where

$\uparrow$   
A=0 Gauge

$U(N)$

(Skew-Hermitian)

$X^i \in \text{Mat}_{N \times N} \in \mathfrak{u}(N)$  (adjoint of  $U(N)$ ),  $i=1, \dots, 9$

$\theta \in \text{Mat}_{N \times N} \otimes (\text{Spin}(9) \text{ rep.})$

$\uparrow$

16 Component spinor

$$g_s = \frac{R}{\ell_s}$$

low energies  $\longleftrightarrow$  "large  $X$ "  $\Rightarrow \text{Tr} [X^i, X^j]^2 = 0 \Rightarrow$  Simultaneously diagonalizable. Diagonal entries of  $X^i \longleftrightarrow i^{\text{th}}$  coordinates of  $N$  D0 branes.

### Decompactification Limit

$$\text{Need } \hat{P}_{11} = \frac{N}{R} \rightarrow \infty$$

$$N \rightarrow \infty$$

$$R \rightarrow \infty$$

$\Rightarrow$

$$N \rightarrow \infty$$

$$\frac{N}{R} \rightarrow \infty$$

Study only processes with states whose energies vanish at least  $\mathcal{O}(\frac{1}{N})$  (other states have  $\infty$ -large energy).

Conclusion:

$$M[\mathbb{R}^{9,1} \times S^1_{\mathbb{R}}] \text{ in IMF} = U(N) \text{ matrix model in the limit that } N \rightarrow \infty$$

DLCQ Refinement (Susskind: hep-th/9704080)

Let  $C_{\mathbb{R}} = \mathbb{R}^{1,1} / \Delta$  as in intro, then

(\*) Total Momentum N Sector of  $M[\mathbb{R}^8 \times C_{\mathbb{R}}]$  = (Quantization of)  $U(N)$  matrix model

$$\mathcal{H}^{M[\mathbb{R}^8 \times C_{\mathbb{R}}]} (\hat{P}_- = \frac{N}{R}) = \mathcal{H}^{U(N)\text{-mat}}$$

↑  
Representations of 11-D Galilean Group w/ 16 SUSY (broken from 32 due to compactification and BPS presence)

DLCQ of Theory  $X_{A_{N-1}}$ : World volume-theory on M5 branes

Same process, but in the presence of N coincident M5 branes:

D4 branes in IIA language  $\longleftrightarrow$  D0-D4 bound states in IMF

Berkovitz & Douglas: 9610236 Fivebranes in M(matrix) Theory:

8-SUSY

Use language of  $N=(1,0)$ ,  $d=6$  SUSY (dimensionally reduce to D0 brane worldvolume

via previous arguments): RHS of (\*) w/  $K$  M5 branes is

$$\mathbb{1} \text{ Hypermultiplet in } \text{Adj}_K + \text{VM of } U(K) + K \text{ Hypermultiplets in Fundamental } K \text{ of } U(N)$$

↑ hypers

Usual M(matrix) Theory (D0-D0) strings

(one Fund. per D0-D4 string)

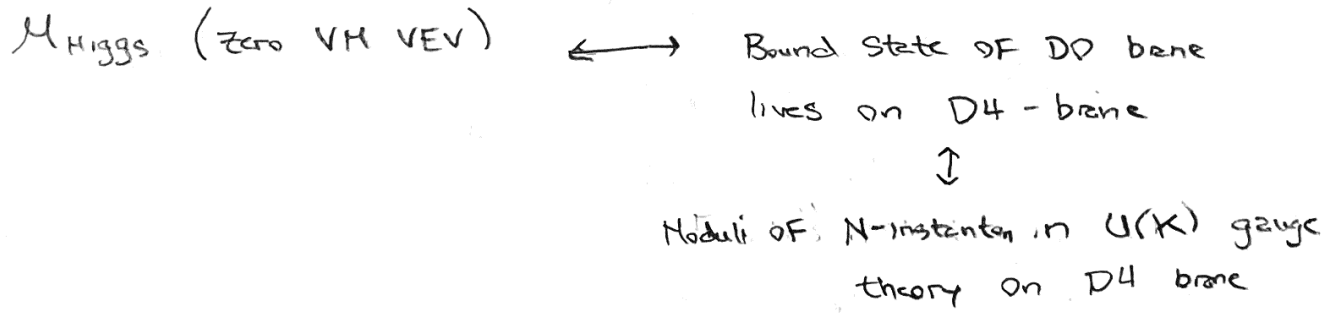
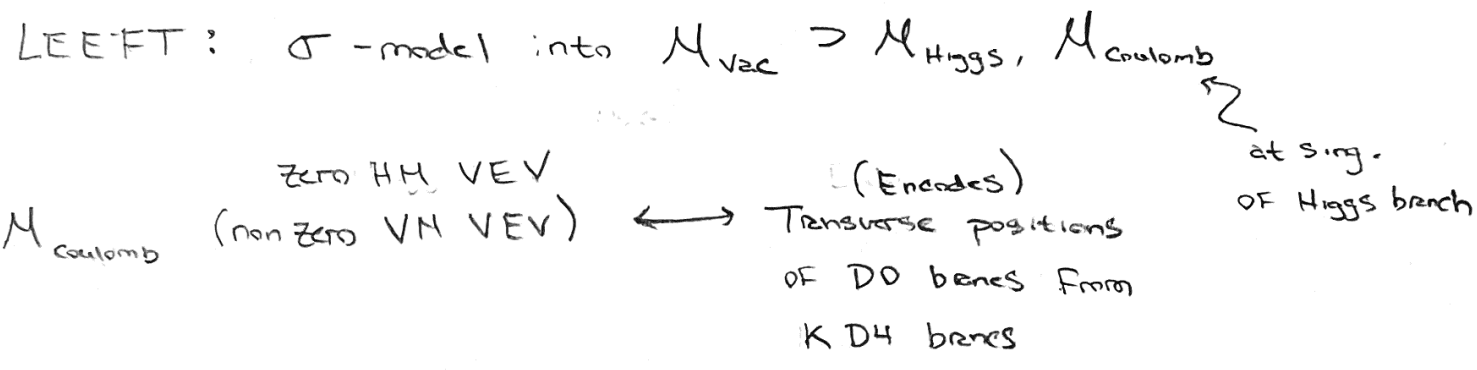
There are 8 Supercharges:

$$8 = \frac{1}{2} \cdot \frac{1}{2} \cdot 32$$

$\uparrow$   
 Restricting to  
 Sector w/  $P_- = \frac{N}{R}$

$\swarrow$   
 BPS M5  
 brane

To get  $X_{A, K+1} [\mathbb{R}^4 \times C_R]$  we look at the low energy effective field theory (should be M5 brane worldvolume theory) in the limit that gravity decouples:  $M_p \rightarrow \infty \Rightarrow l_p \rightarrow 0 \Rightarrow g_s = g_{QM} = M_p^6 R^3 \rightarrow \infty$ .



$g_{QM} \rightarrow \infty$  limit: Higgsed VM gain infinite masses  $\Rightarrow \mathcal{M}_{Coulomb}$  "decouples" from the interior of the Higgs branch

Metric on  $\mathcal{M}_{Higgs}$ : independent of  $g_{QM}$

- Gains no Quantum Corrections (non-renorm. Thm: hep-th/9603042 Argyres, Plesser, Seiberg)

$\mathcal{M}_{\text{Higgs}}$ : Let adjoint HM Scalars be:  $X, \tilde{X} \in \text{Mat}_{N \times N} \mathbb{C}$

HM Scalars:  $q^i \in \mathbb{N}$   
 $\tilde{q}^i \in \overline{\mathbb{N}}$

Then

$$\mathcal{M}_{\text{Higgs}} = \left\{ \begin{array}{l} [X, X^\dagger] - [\tilde{X}, \tilde{X}^\dagger] + q_i q_i^\dagger - (\tilde{q}^i)^\dagger \tilde{q}^i = 0 \\ [X, \tilde{X}] + q_i \tilde{q}^i = 0 \end{array} \right\}$$

ADHM

$$\mathcal{M}_N[U(K)] \cong \mathbb{R}^4 \times \mathcal{M}_N^0[U(K)]$$

Param. by  $\text{Tr}(X), \text{Tr}(\tilde{X})$

$\dim_{\mathbb{C}} \mathcal{M}_H = 4NK$  (HK manifold)

Quantization Gives:

$$\hat{P}_+ = \hat{H} = \frac{R}{N} g_{\mu\nu} \hat{\Pi}_\alpha \hat{\Pi}_\beta + \text{Form } \alpha, \beta \in 1, \dots, 8NK$$

$$\hat{P}_- = \frac{N}{R} = \frac{1}{\Delta P_-} g_{\mu\nu} \hat{\Pi}_\alpha \hat{\Pi}_\beta$$

Included For Convenience ( $\hat{H} = \hat{P}_+ = \frac{P_i^2 + M^2}{P_-}$ )

Rotations  $\mathbb{R}$  to S-brane

Global Symmetries:  $SU(2)_R \times SU(2)_L \times \text{Spin}(5) \times U(K)_{\text{global}}$  R-sym of 6D Theory x

Rotation Symmetries inside S-brane and  $\mathbb{R}$  to Light-cone  
 Good - (Spin(4)).

Bosonic Space-time Symmetries preserving  $\hat{P}_- = \frac{N}{2}$  sector: (commuting with  $\hat{P}_-$ )

Galilean generators  $M_{ij}$ ,  $P_i$ ,  $H = P_+ = P_0 + P_1$ ,  $V_i = M_{0i} - M_{1i}$   
 ↑ rotations      ↑ translations      ↑ boosts

Preserved Conformal Gens:  $K_- = K_0 - K_1$ ,  $T = D - M_{01}$   
 ↑ Special Conf.      ↑ Dilation      ↑ Boost in  $x'$  direction

Centralizer  $SO(2,6) [\hat{P}_-]$

In our model

$K, T =$  Combinations of derivatives of Kähler potential  $K$  and  $\hat{\Pi}$ -ops.

Where

$$K = \text{Tr} \left[ \varrho_i^+ \varrho_i + (\tilde{\varrho}^i)^+ (\tilde{\varrho}_i) + X^+ X + \tilde{X}^+ \tilde{X} \right]$$

Fermionic: • 8 spacetime SUSY "Q"s

• 8 superconformal generators "S"s

Need for resolution of singularities: Want to make sense of

State  $\leftrightarrow$  Op. Correspondence

but all primary states:  $(K_- |S\rangle = 0$ , are localized at the origin:

a singular point. Want to make sure there are no hidden degrees of freedom at the origin.

Resolution: • Add FI term (QFT P.O.V.)



Turn on 3-Form Field (M-theory) in 11D.

Resulting Theory is not Theory X, but Flows to it in the IR.

↑  
not even Lorentz invariant in 11D

(FI param  $\xi$  acts as UV cutoff.)

• Did some Computations For cases  $N=1, K=2, N=2, K=1$

where  $\mathcal{M}_{1+} = \mathbb{R}^4 \times \mathbb{R}^4 / \mathbb{Z}_2$ , Resolution is usual  
blowup of  $\mathbb{Z}_2$  singularity.

Refs: 'Aharony - Berkooz - Kachru - Seiberg - Silverstein : hep-th/9707079

• AB-Seiberg : 9712117

• Banks-Fischler-Shenker-Susskind : 9610043

• Susskind : 9704080

• Berkooz-Douglas : 9610236

• Witten : 9510135